87 RM 27 Hm (N)

2014

HIGHER MATHEMATICS

Full Marks – 80

Pass Marks – 20

Time : Three hours

*Attempt **all** questions.*

The figures in the right hand margin indicate full marks for the questions.

For Question Nos. 1 to 5, write the letter corresponding to the correct answer.

1. The harmonic mean between a and b is 1

(A) $\frac{a+b}{ab}$

(B) $\frac{ab}{a+b}$

(C) $\frac{a+b}{2ab}$

(D) $\frac{2ab}{a+b}$

2. The coefficient of x^4 in the expansion of $\left(x-\frac{1}{x}\right)^{10}$ is 1

(A) –120

(B) 120

(C) –210

(D) 210

3. If $A=\begin{bmatrix}-1 & -1\\ k & 2\end{bmatrix}$ and $A^2 = A$, then the value of k is 1

(A) 0

(B) 1

(C) 2

(D) –1

4. The value of tan (– 480°) is 1

(A) $-\sqrt{3}$

(B) $\sqrt{3}$

(C) $-\frac{1}{\sqrt{3}}$

(D) $\frac{1}{\sqrt{3}}$

5. The angle between two equal forces P and P when their resultant is also equal to P, is 1

(A) 60°

(B) 45°

(C) 120°

(D) 90°

6. Is addition a binary operation on the set of all odd integers ? Give reason for your answer. 1

7. If P(n) is the statement "n^2+2 is divisible by 3", show that P(7) is true. 1

8. Define the transpose of a matrix. 1

9. If A and B are symmetric (of the same order), show that A – B is symmetric. 1

10. Define a "reciprocal expression". 1

11. Find the angles in the range $-360° < \theta < 360°$ coterminus with 60°. 1

12. When are forces acting on a body said to be in equilibrium ? 1

13. What are the resolved parts of a force of 50 kgwt, if the inclination of the force to one of the resolved parts is 30° ? 1

14. Prove that the binary operation * on $\mathbb{Z}$ defined by $a * b = a + b - 3$, is associative. 2

15. Find which term of the G P : 9, 3, 1, ... is $\frac{1}{243}$. 2

16. Find the middle term in the expansion of $(x+y)^6$. 2

17. Find 3A – 2B when $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$ 2

18. If $A + B + C = 180°$ prove that $\tan\left(A + \frac{B}{2}\right) = \cot\left(\frac{C-A}{2}\right)$ 2

19. Find the sum of the first n terms of a GP where first term and common ratio are a and r respectively. 3

20. If $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 3 \\ 3 & -2 \end{bmatrix}$, show that AB = AC although $B \neq C$. 3

21. Prove the identity : $27(x+y+z)^3 - (x+2y)^3 - (y+2z)^3 - (z+2x)^3$
$= 3(x+3y+2z)(2x+y+3z)(3x+2y+z)$ 3

22. If $a + b + c = 0$, prove that 3
$b^2 + bc + c^2 = -(bc + ca + ab)$

23. Solve for θ $(0° < \theta < 360°)$: $\tan^2\theta + \cot^2\theta = 2$ 3

24. If three forces acting at a point be such as can be represented in magnitude, direction and sense by three sides of a triangle taken in order, then prove that the forces are in equilibrium. 3

25. Construct the composition table for the set $S = \{1, 2, 3, 4, 5, 6\}$ with respect to the binary operation of multiplication modulo 7. From the table find the identity element and the inverse of each element of S. 4

26. Prove by Mathematical Induction that, for every $n \in \mathbb{N}$, 4
$$\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

Or

$3^{2n} - 1$ is divisible by 8.

27. Prove that, every square matrix can be expressed in one and only one way, as the sum of a symmetric matrix and a skew symmetric matrix. 4

28. State and prove Binomial Theorem for a positive integral index. 5

29. Factorise : $2x^6 - 3x^5 - 3x^4 + 3x^2 + 3x - 2$ 5

Or

Resolve into two quadratic factors : $x^4 - 7x^3y + 14x^2y^2 - 14xy^3 + 4y^4$

30. Find the trigonometric ratios of $(180° + \theta)$ in terms of those of θ. 5

31. The digits of a three-digit number are in AP and their sum is 15. The number obtained by reversing the digits is 594 more than the original number. Find the number. 6

32. Forces of magnitude P, 2P, 3P, 4P, 5P respectively act at the angular point A of a regular hexagon ABCDEF towards the other angular points taken in order. Show that the magnitude of the resultant is $2\sqrt{19+10\sqrt{3}}$ P and $\tan\theta = \frac{5+4\sqrt{3}}{\sqrt{3}}$ where θ is the angle which the resultant makes with AB. 6

Or

The resultant of forces P and Q is R ; if Q is doubled, R is doubled and if Q is reversed, R is again doubled. Show that $P^2 : Q^2 : R^2 = 2 : 3 : 2$.